ANITA Summer School – Cosmic Explosions
Core-Collapse Supernovae

You will find that some of these problems don't contain all the information that you need to solve them. That's intentional: Filling in the dots by using the literature or an informed guess based on something else that we've covered in the lecture is part of the exercises. The point is not so much to get the correct equations and numerical results, but to discuss and justify the assumptions that you make. You shouldn't necessarily expect that you'll be able to work through all of them within 90 minutes. Pick the ones that seem most interesting to you.

1. MHD-Driven Hypernovae – Requirements

Most scenarios for obtaining explosion with energies $\gg 10^{51}$ erg rely on tapping the rotational energy of the supernova core to create strong magnetic fields that ultimately power the explosion (e.g. by creating jets). This requires rapidly rotating progenitor cores, especially if the neutron star is to survive in the explosion as in the millisecond magnetar model:

(a) Estimate the required spin rate of a neutron star for reaching a rotational energy $E_{\mathrm{rot}} = I\omega^2/2 = 10^{52}$ erg (where I is the moment of inertia and ω is the angular velocity).

(b) Using conservation of angular momentum ($L = I\omega$), infer the required rotation rate in the progenitor. You can assume that the matter that makes up the neutron star is initially located within a radius of $\sim$1000 km.

2. Light Curves and Spectra – Simple Estimates

Let us consider Type IIP supernovae from red supergiants, and assume a typical explosion energy $E \sim 10^{51}$ erg and ejecta mass $M \sim 12\,M_\odot$.

(a) We expect that the width of the line features will roughly reflect the ejecta velocity (though the details of line formation are somewhat complicated). Estimate the typical ejecta velocity and find some spectra of Type IIP supernovae to check whether your estimate is reasonable.

(b) During the plateau phase of Type IIP supernovae, the photon luminosity feeds mostly on the thermal energy from shock heating. Let us form a crude estimate of the plateau luminosity – note that there are more much more sophisticated ways to do this.

First, we need the internal energy E_{therm} of the ejecta at shock breakout. How do you expect this to be related to the explosion energy?

(c) Calculate the typical (average) temperature T_0 of the shock H envelope at shock breakout assuming that the ejecta are in the radiation-dominated regime (i.e. the internal energy per unit volume is $u = aT^4$). You'll have to make simplifying assumptions about the temperature distribution of the ejecta. Effectively, you will get a value for a shell roughly in the middle of the H envelope.

(d) You should find that T_0 is considerably higher than the recombination temperature T_{rec}. Before we form a recombination front that propagates down through the hydrogen envelope, the temperature needs to drop considerably. Let us assume that this happens by adiabatic cooling (though radiative cooling by diffusion also plays an important role in practice). How much does the photosphere need to expand until roughly half of the envelope has dropped below the recombination temperature? You can assume that the ejecta expand self-similarly.

(e) Using your estimate for the photospheric radius R_{phot} during the plateau, estimate the luminosity using the Stefan-Boltzmann law. Note that you will not recover the exact scaling law from the lecture, because this would require a more sophisticated derivation including effects of radiative diffusion.

Check whether the predicted value roughly agrees with observed Type-IIP plateau luminosities.

3. Neutrino Trapping

The dominant scattering process for neutrinos during iron core collapse is neutrino-nucleus scattering $\nu + A \to \nu + A$. The scattering opacity depends on the number density n_A of nuclei, their mass and charge number (A and Z), and the neutrino energy ϵ,

$$\kappa_{\mathrm{s}} \approx \frac{\sigma_0}{32}\left(\frac{\epsilon}{m_e c^2}\right)^2 A^2 n_A \left[C_A - C_V + (2 - C_A - C_V)\left(\frac{2Z - A}{A}\right)\right]^2,$$

where the Fermi constant for weak interactions is hidden in $\sigma_0 = 1.761 \times 10^{-44}$ cm^2, and $C_V = 0.96$ and $C_A = 1/2$ are vector and axial-vector coupling constants. We shall use this scattering opacity to estimate when neutrino trapping occurs during collapse:

(a) What are reasonable values for Z and A? Assuming that the average energy of escaping neutrino is 5 MeV, express κ_{s} as a function of ρ.

(b) Neutrinos trapping occurs roughly when the mean free path ($\lambda = 1/\kappa_{\mathrm{s}}$) equals the radius R of the collapsing core. Estimate R and then solve $\kappa_{\mathrm{s}} R = 1$ for the trapping density.

4. Neutrino Mean Free Path and Equilibration Time

Neglecting a few blocking factors and assuming neutrino energies ϵ much larger than the proton-neutron mass difference, the cross section for absorption of electron neutrinos by neutrons (or electron antineutrinos by protons) is roughly,

$$\sigma \simeq \frac{\sigma_0 \epsilon^2}{4 m_e^2 c^4}(g_V^2 + 3 g_A^2),$$

where $g_V = 1$ and $g_A = 1.254$.

(a) From σ, we obtain an opacity for neutrino absorption as $\kappa = \sigma n_n$, where n_n is the neutron number density (Check its dimensions). Estimate the mean free path $1/\kappa$ and the equilibration time-scale $t_{\mathrm{eq}} = 1/(\kappa c)$ for a neutrino energy of $\epsilon = 100$ MeV and a density of 4×10^{14} g cm^{-1} (How can you estimate n_n from this?).

(b) The primary detection channel for supernova neutrinos in water Cherenkov detectors is $p + \bar{\nu}_e \to n + e^+$, where the hydrogen nuclei in the water molecules are the proton targets. Using the water mass of 50 kT in Super-Kamiokande, the neutron star binding energy radiated in neutrinos in a supernova ($\sim 3 \times 10^{53}$ erg), and an average electron antineutrino energy of 15 MeV, calculate the number of detection events in Super-K expected from a supernova in the Large Magellanic Cloud at a distance of 50 kpc (1 kpc $= 3.086 \times 10^{21}$ cm). Assume that roughly $1/6$ of the neutrinos come out as $\bar{\nu}_e$.